Description of various parameters in **Altimeter LCS-Cores and Stretching Directions (Application to Oil-Spill trajectory Nowcasting and Forecasting)**

Lagrangian Coherent Structure Cores (LCS -Cores)

Lagrangian Coherent structures(LCS) arise in Ocean due to non-linear dynamics of Ocean. These 2-D structures have an ability to facilitate or block the material transport (of seawater + passive tracers) through them, thus organizing the flow pattern of passive tracers in the ocean. Its computation involves the 2-D advection of particles (starting from a well-defined grid) at a current time with altimeter velocities to a certain period of time (15 days in this case). Details can be found in Farazmand et al. 2012 and Onu et al. 2015. If the advection is carried out backward in time, then we can calculate the `attracting LCS`. These attracting LCS's form the flow pattern of the passive tracer. These LCS's can be termed as 'skeleton of flows' for a passive tracer. Passive tracer can be a Lagrangian float near surface, **Oil Spills, Chlorophyll**, occasionally **SST** and **Salinity**. **LCS-Cores** are the strongest of the LCS and these will last longer in time relative to other LCS's. LCS-Cores strongly affect the passive tracers over a larger period to time. Typical time-scale of these LCS-Cores can be few days. Details of computation of LCS-Cores can be found in Olascoaga et al. 2012. Strength of the LCS-Core is determined by the average lagrangian strain-rate (unitless quantity). These LCS Cores form fingering type instabilities along the **stretching directions** for an oil spill patch as illustrated in the below graphic. Other LCSs also affect the oil spill patch shapes but the LCS Cores are the most prominent among them.

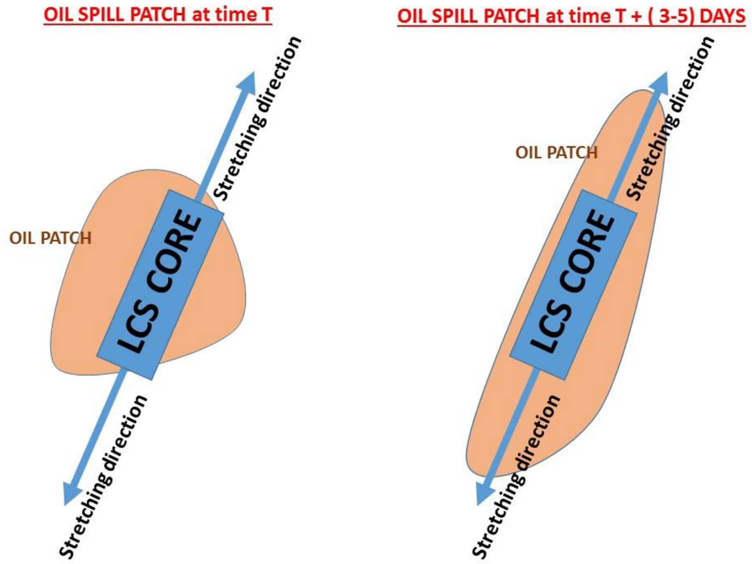


Figure 1. Illustration of stretching of an oil-patch if it happens to be on the LCS Core. This is a fingering type instability.

In the GUI, we have presented the **LCS-Cores**. These have been calculated form the altimeter velocity field. Duration of advection is 15 days.

LCS- Cores are shown as Scattered points in the plots. The color coding shows their strength (unit-less).

Stretching directions

As discussed in the previous section, the stretching directions are the directions associated with the LCS-Core. It is computed as the eigenvectors of the Cauchy-Green Tensor obtained through advection of particles. In physical sense, it means the direction of maximum relative stretching. **Figure 1** shows the impact of stretching directions. We expect the passive tracers to align to these directions.

In the GUI, the arrow direction corresponds to the **stretching direction** of the LCS.

Background Field – FiniteTime Lyapunov Exponent (FTLE)

This scalar quantity is an intermediate field coming in calculation of LCS and LCS-Cores. This is a quantity that characterizes the rate of separation of infinitely close trajectories (of advected particles backward in time). Higher values signify relatively larger rate of separation than its neighboring locations. It is computed by advecting the particles for a particular time (15 days in this case) and finding the highest eigenvalue of the Cauchy green tensor that arise from advection. Details of computation can be found in Farazmand et al. 2012.

Ridges of FTLE forms generally contains LCS although it is not the sufficient condition. But it provides a nice picture of the overall structures present in the ocean arising from velocity field for a particular day.

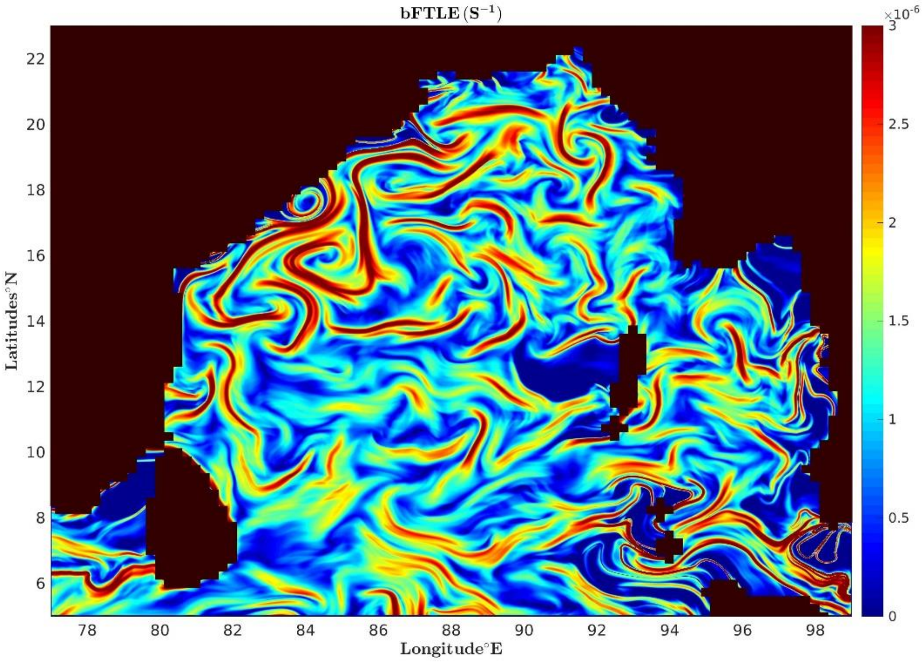


Figure 2. Backward time FTLE filed for a particular day (4th March 2016) in the Bay of Bengal calculated using the altimeter velocity field.

In the GUI, background is the FTLE field in (day $^{-1}$). Color coding is in greyscale.

Methodology

Differential equation for the two dimensional incompressible fluid motion is

$$\dot{\boldsymbol{x}} = \boldsymbol{v}(x,t)\,, \qquad \nabla . \boldsymbol{v} = 0 \tag{1}$$

where the $\dot{x}$ represents the differential rate with respect to time and ∇ is the gradient with respect to variable x, and the velocity field $\boldsymbol{v}(x,t)$ denotes the time and location dependent velocity field (from altimeter) of the fluid.

For calculating the Lagrangian Coherent structures in two dimensions the following algorithm is given by Farazmand et al., 2012 and Onu et al. 2015.

In order to calculate the strength of LCSs we the $r(\boldsymbol{x}_t, t)$ parameter as given in Olascoaga et al. 2012.

The instantaneous normal attraction rate of an LCS at point $\boldsymbol{x}_t$ with unit normal $\boldsymbol{n}_t$ is measured by the Lagrangian strain rate

$$r(\boldsymbol{x}_t, t) = \langle \boldsymbol{n}_t, S(\boldsymbol{x}_t)\boldsymbol{n}_t \rangle \tag{2}$$

where $\boldsymbol{S} = \frac{1}{2}[\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^*]$ is the rate-of-strain tensor.

If the value of $r(\boldsymbol{x}_t, t)$ is negative at all times, then we say that these point are LCS-Cores. The averaged value of $|r(\boldsymbol{x}_t, t)|$ is the measure of strength of the LCS-Core.

Different Layers in GUI

1) FTLE layer:

Sample FTLE layer shown in **Figure 3**.

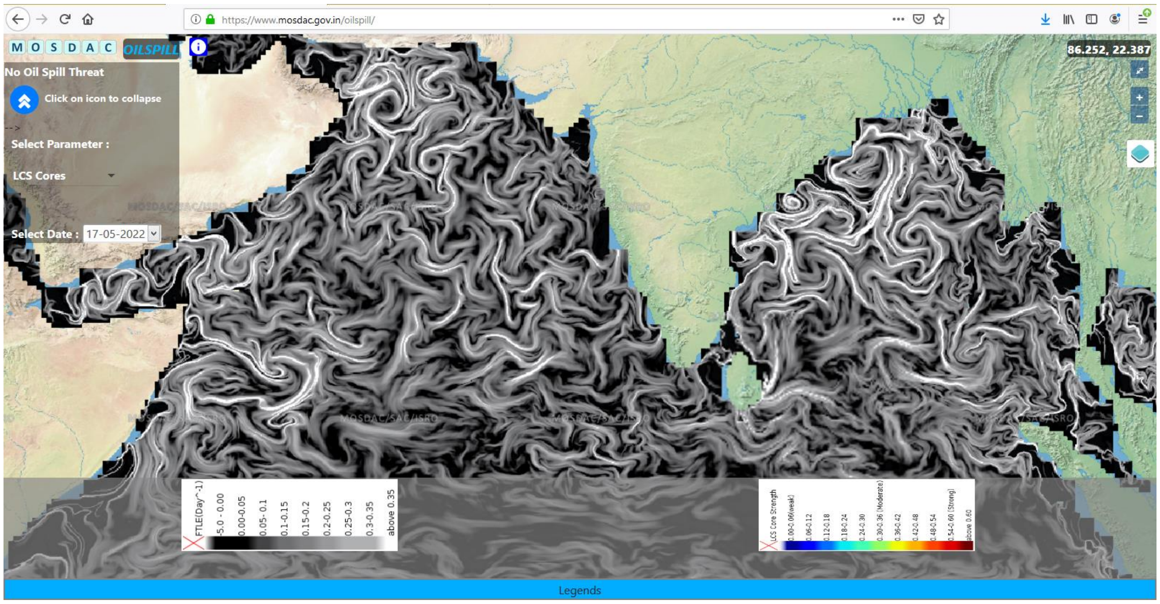
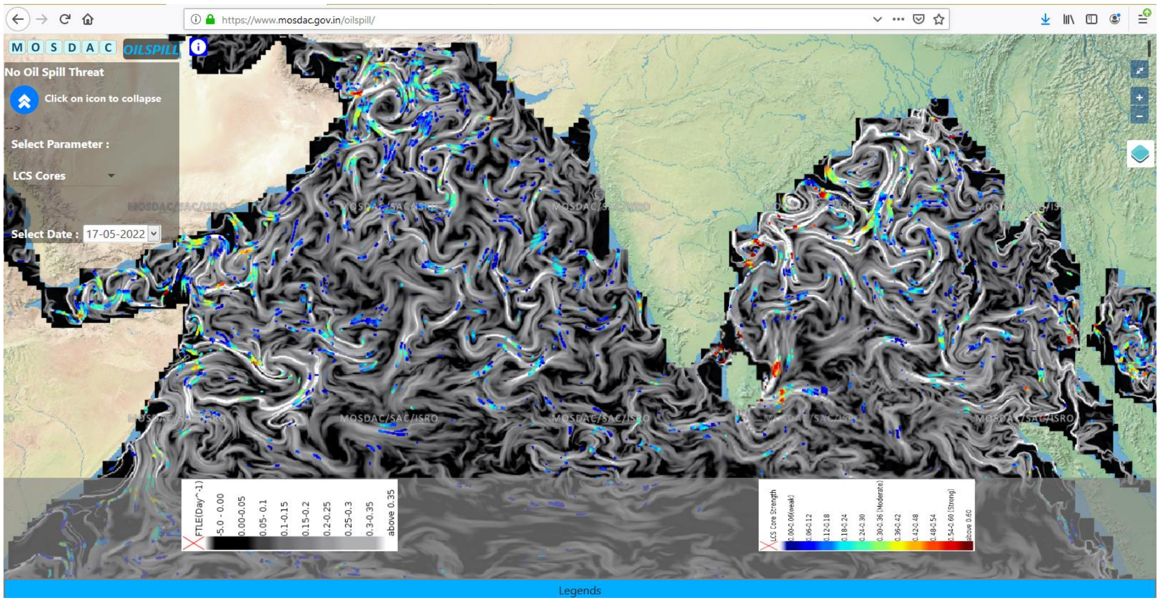


Figure 3. Sample FTLE layer of GUI at MOSDAC

2) LCS-Core layer:

Sample LCS-Core layer shown in **Figure 4** overlaid over FTLE layer.



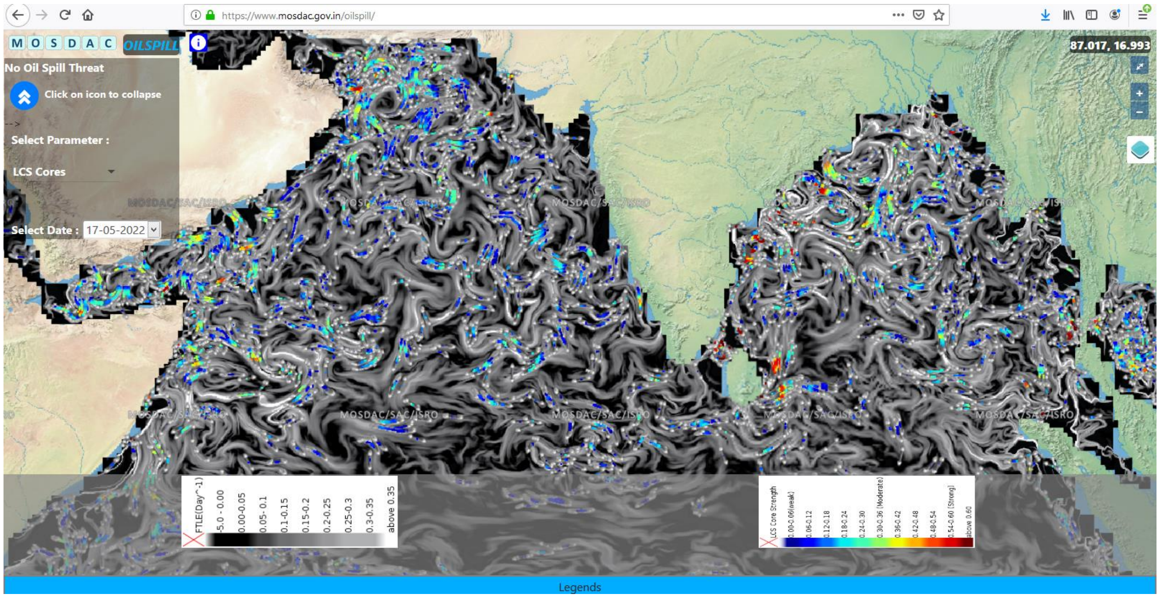
Select Parameter : LCS Cores

Select Date : 17-05-2022

Figure 4. Sample FTLE + LCS-Core layer of GUI at MOSDAC

3) Stretching-directions layer

Sample LCS-Core layer shown in **Figure 5** overlaid over (FTLE + LCS-Core) layer



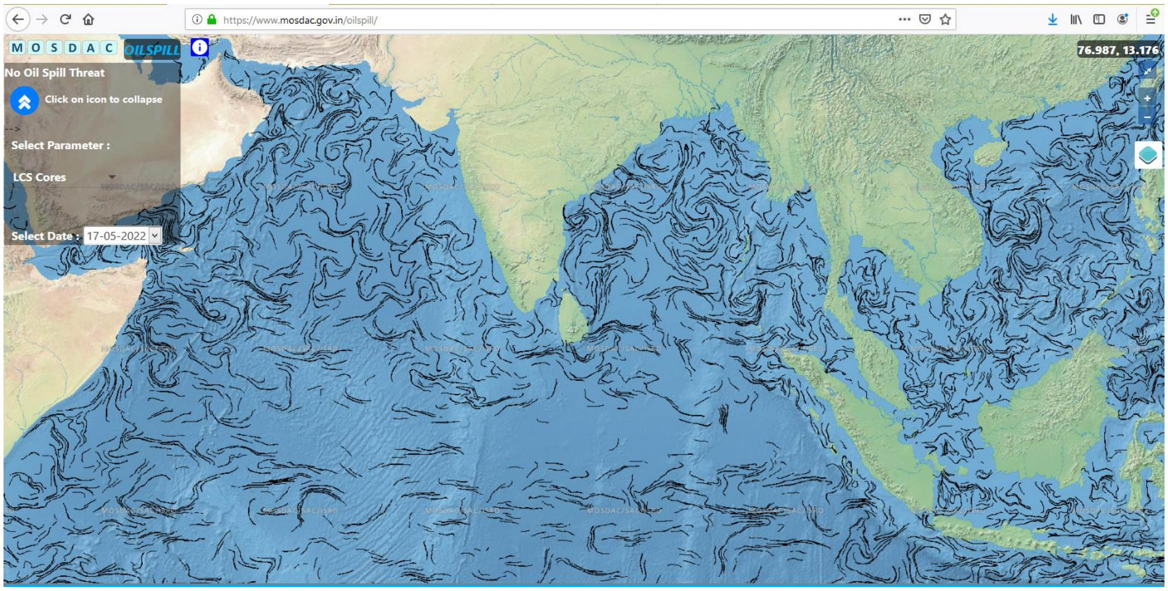
Select Parameter : LCS Cores

Select Date : 17-05-2022

Figure5. Sample FTLE + LCS-Core layer + Stretching Directions layer of GUI at MOSDAC

4) LCS layer

Sample LCS layer shown in **Figure 6**.



Select Parameter : LCS Cores

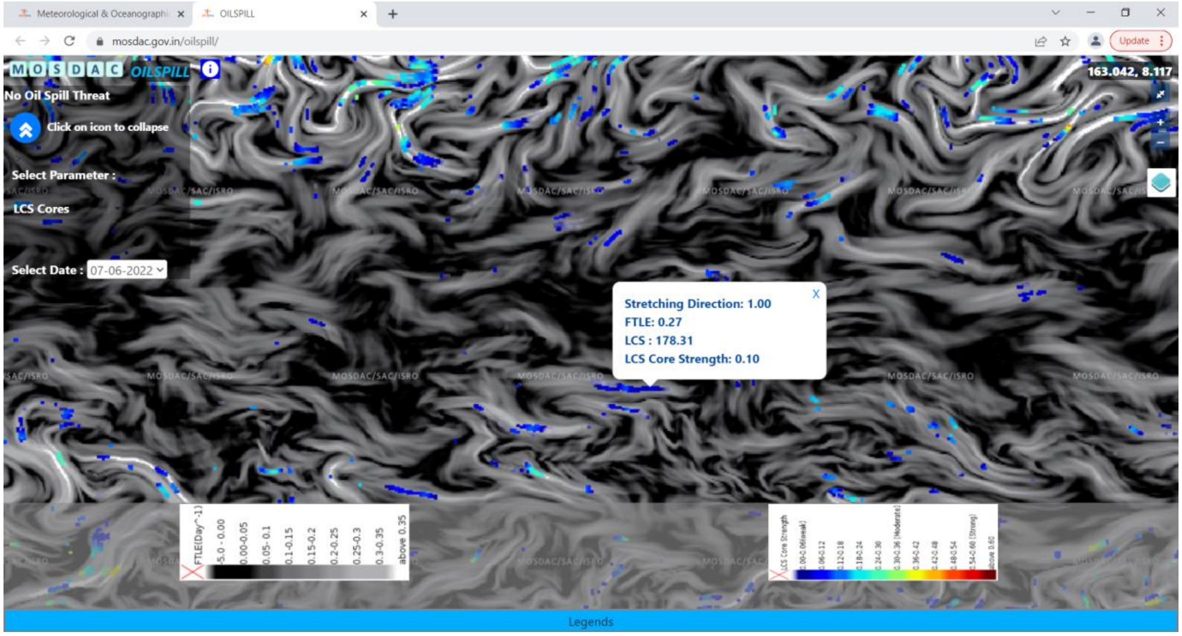
Select Date : 17-05-2022

Figure 6. Sample LCS layer of GUI at MOSDAC

5) Other options available:

i) Point probing implemented in GUI:

Point probe is also implemented to know the quantitative value of the layers. Point probing is shown as a sample in **Figure 7**.



Select Parameter : LCS Cores

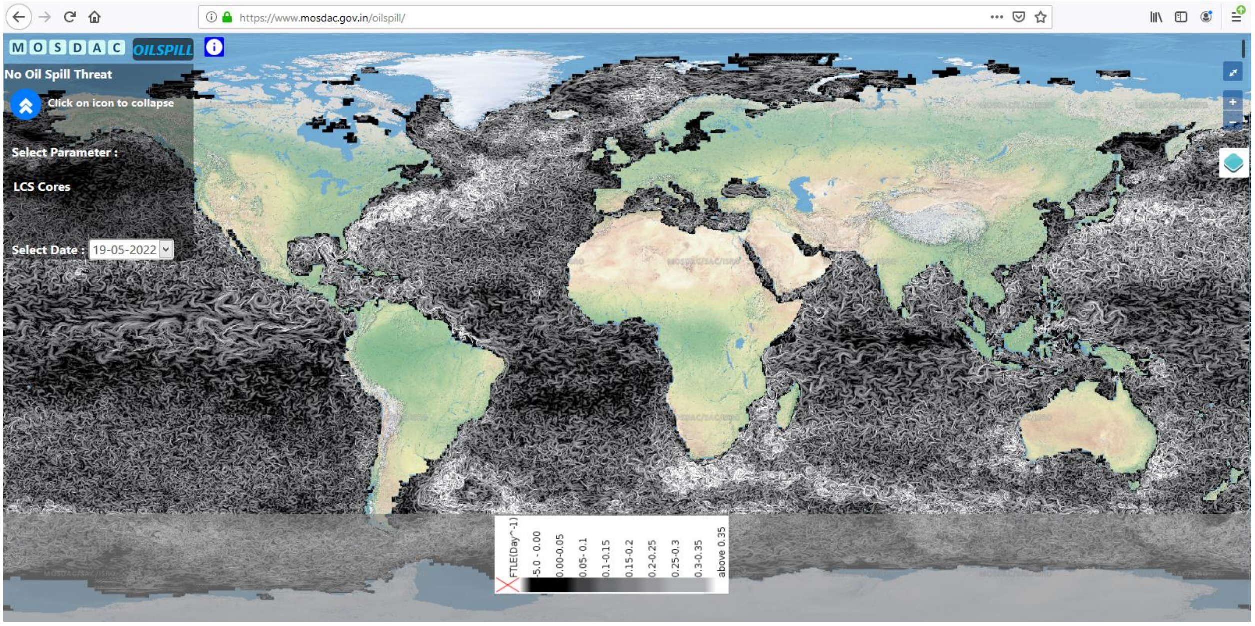
Select Date : 07-06-2022

Stretching Direction: 1.00
FTLE: 0.27
LCS : 178.31
LCS Core Strength: 0.10

Figure 7. Zoom + point probe over GUI at MOSDAC

ii) Some Sample plots form GUI:

Figure 8 and **Figure 9** show some layers on global scale. **Figure 10** and **Figure 11** show the different layers for Gulf Stream region of ocean.



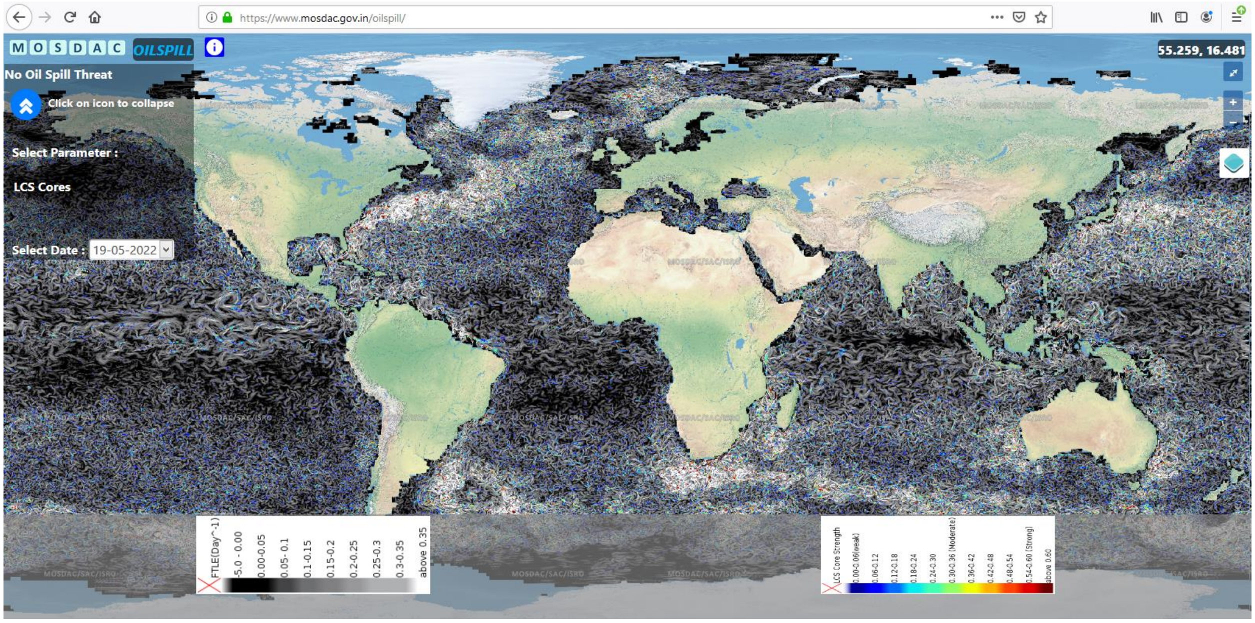
https://www.mosdac.gov.in/oilspill/

No Oil Spill Threat

Select Parameter : LCS Cores

Select Date : 19-05-2022

Figure 8. Global FTLE layer of GUI at MOSDAC



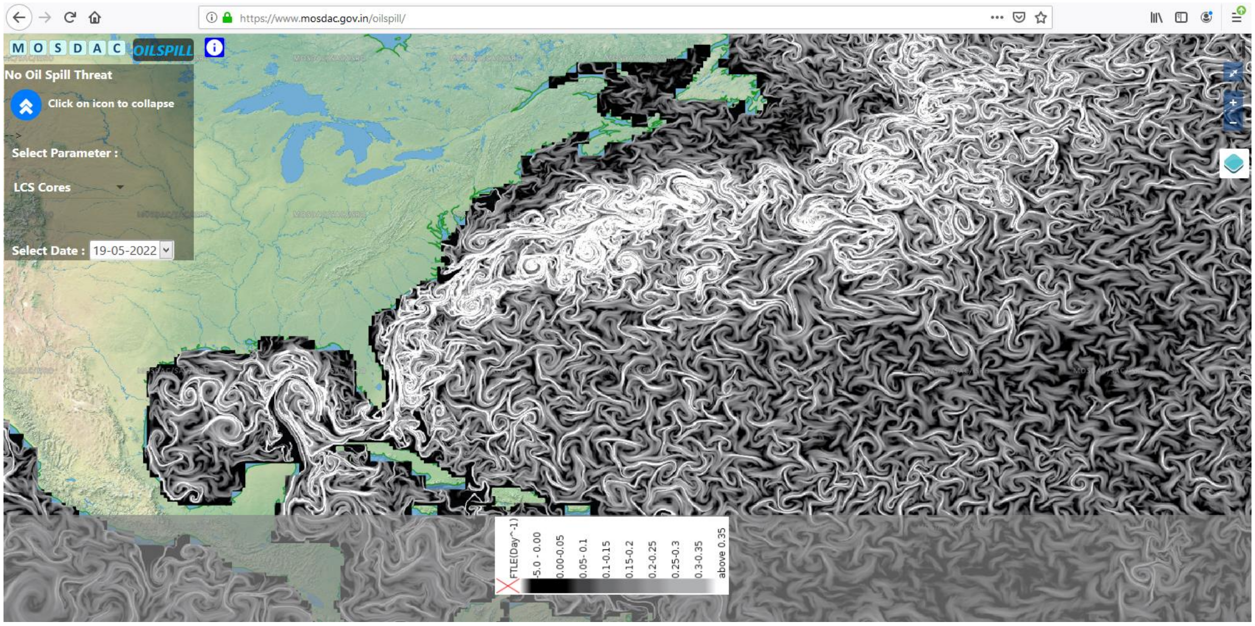
https://www.mosdac.gov.in/oilspill/

No Oil Spill Threat

Select Parameter : LCS Cores

Select Date : 19-05-2022

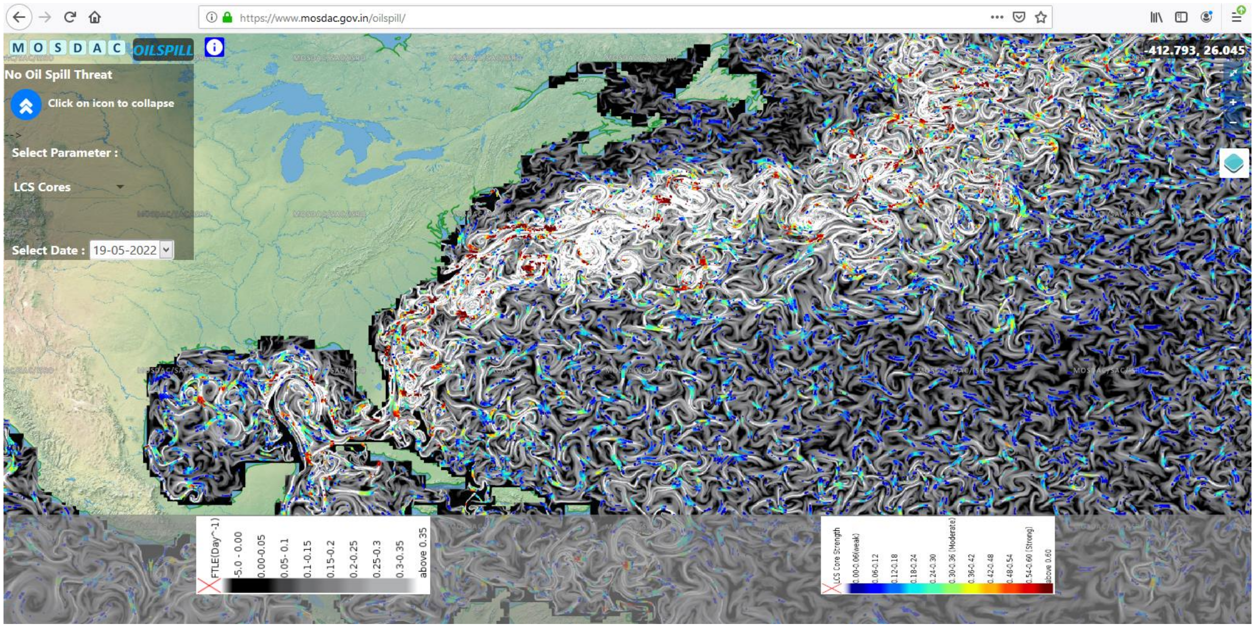
Figure 9. Global FTLE + LCS Cores layer of GUI at MOSDAC



Select Parameter : LCS Cores

Select Date : 19-05-2022

Figure 10. Gulf Stream FTLE layer of GUI at MOSDAC



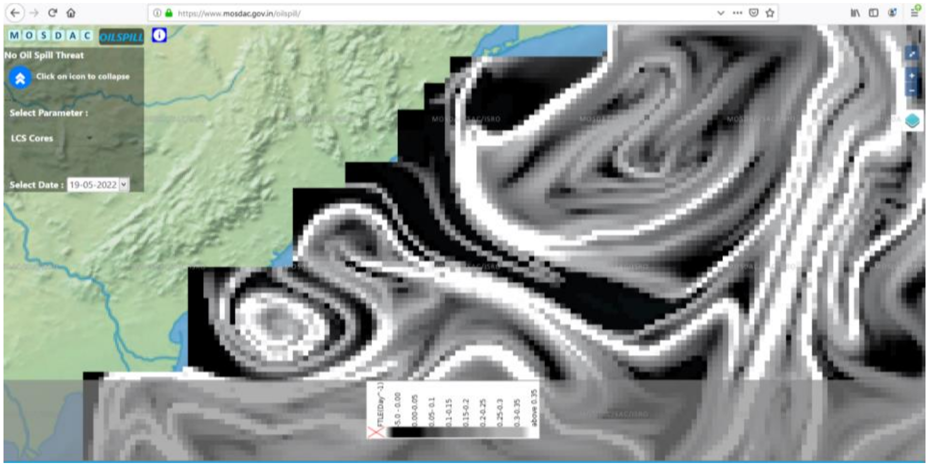
Select Parameter : LCS Cores

Select Date : 19-05-2022

Figure 11. Gulf Stream FTLE + LCS Cores layer of GUI at MOSDAC

Interpretation of GUI

FTLE Layer: FTLE layer is a two dimensional field that contains the LCS within and forms a contiguous field that effects the passive tracers like oil-spills, Chlorophyll and occasionally SST/SSS, red tides, etc. FTLE aligns with SST/Chlorophyll gradient fields.

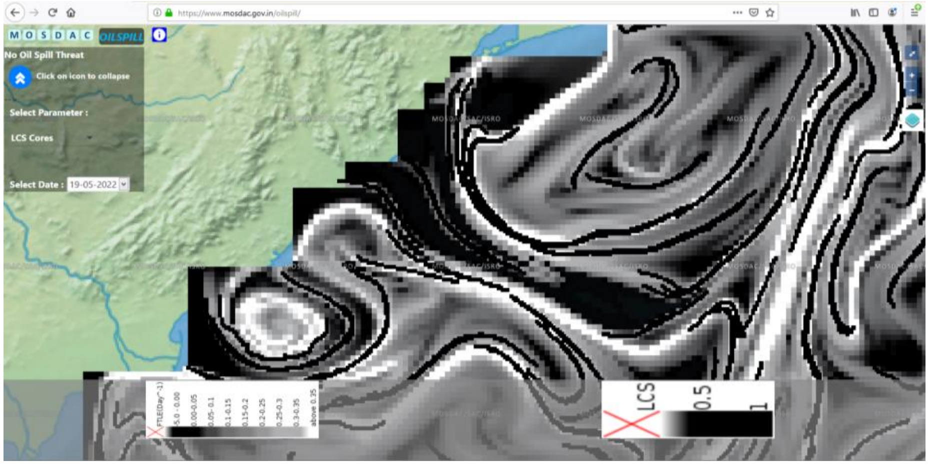


Select Parameter : LCS Cores

Select Date : 19-05-2022

Figure 12. Snapshot of FTLE layer of GUI at MOSDAC

LCS layer: As discussed in section 2.5 LCS layer are curves (ref. black curves **Figure 13**) that have a strong impact on the trajectories of the passive tracers. LCS layer is a stronger indicator than the FTLE field.

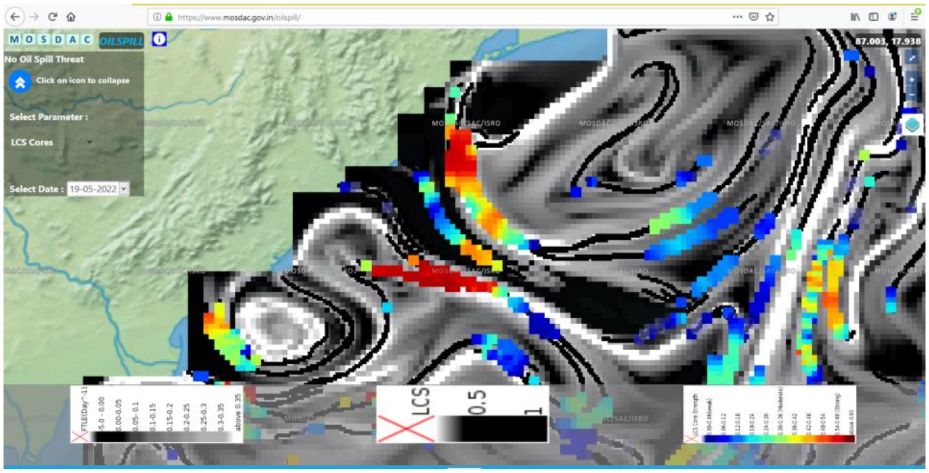


Select Parameter : LCS Cores

Select Date : 19-05-2022

Figure 13. Snapshot of FTLE layer (background) + LCS layer (black curves) of GUI at MOSDAC

LCS Core layer: As discussed in section 2.2 LCS-Core regions are the strongest of LCS regions (ref. colored scatter in **Figure 14**); Their strength is indicated in color-code with Red color indicating relatively higher strength and Blue color representing relatively weaker LCS-Cores. LCS-Cores are even a stronger indicator than LCS and have a predictive capability of few days (Olascoaga and Haller 2012). LCS have relatively lesser predictive capability than LCS.



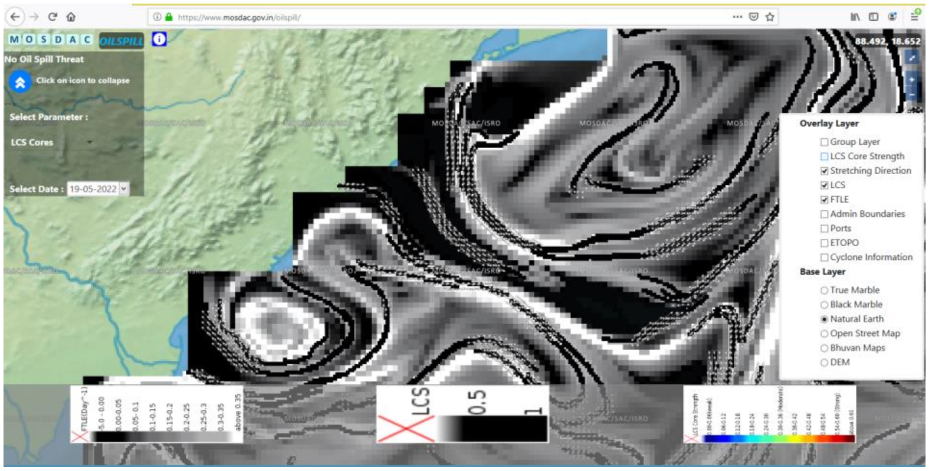
No Oil Spill Threat

Select Parameter : LCS Cores

Select Date : 19-05-2022

Figure 14. Snapshot of FTLE layer (background) + LCS layer (black curves) + LCS-Cores (colored scatter) of GUI at MOSDAC

LCS Stretching directions: Stretching directions (ref. white vectors **Figure 15**) will provide the preferred directions of advection/stretching for passive tracers near the LCS-Core regions.



No Oil Spill Threat

Select Parameter : LCS Cores

Select Date : 19-05-2022

Overlay Layer
- ☐ Group Layer
- ☐ LCS Core Strength
- ☑ Stretching Direction
- ☑ LCS
- ☑ FTLE
- ☐ Admin Boundaries
- ☐ Ports
- ☐ ETOPO
- ☐ Cyclone Information

Base Layer
- ○ True Marble
- ○ Black Marble
- ● Natural Earth
- ○ Open Street Map
- ○ Bhuvan Maps
- ○ DEM

Figure 15. Snapshot of FTLE layer (background) + LCS layer (black curves) + Stretching directions layer of GUI at MOSDAC

Summarizing, in terms of impact on the passive trajectories/fields the orders is shown as:

FTLE < LCS < LCS-Cores

LCS-Cores have the most predictive capability for nowcasting trajectories of passive tracers.

These GUI can be a helpful tool for analyzing and predicting the fields as well as problems where surface advection plays an important role in trajectory/field evolution.

1) Oil-spill trajectory nowcasting

2) Chlorophyll field (formation and evolution)

3) SST field (formation and evolution)

4) Red Tides (formation and evolution)

5) Trajectory planning in the Ocean

6) Waste Disposal problems in the ocean

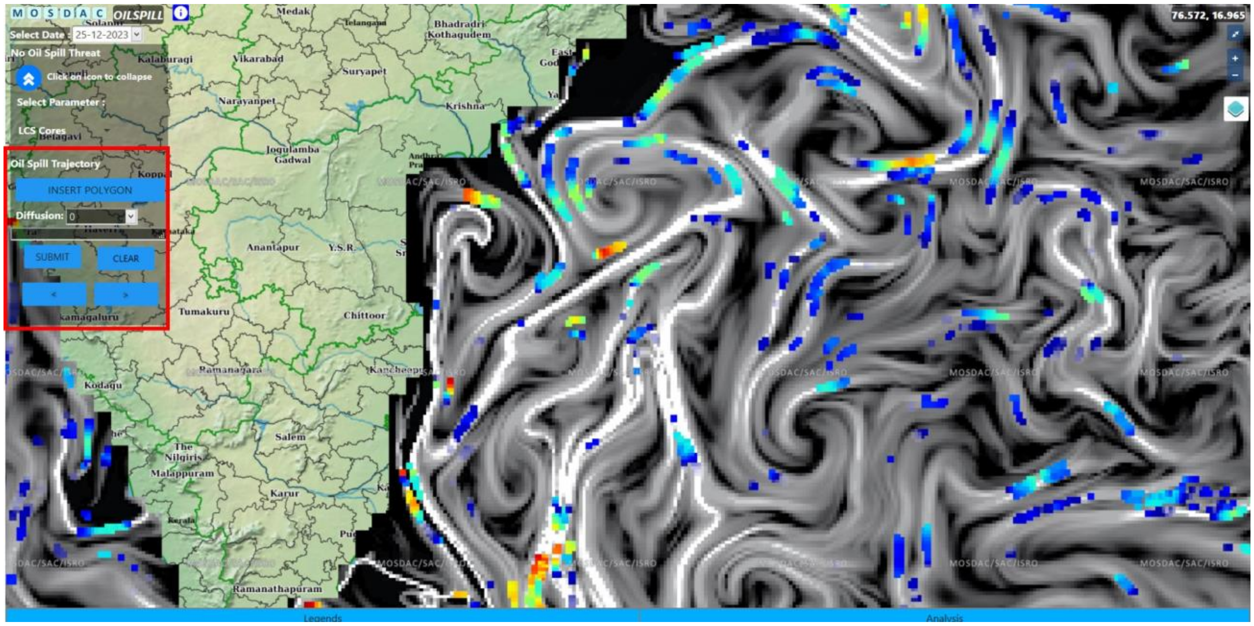
Newly Added Component (December 2023):

Oil Spill trajectory forecast tool

- This tool provides oil spill trajectory forecast over user provided area for next 5 days
- Uses Lagrangian method for forecasting oil-spill trajectories.
- User defined lateral diffusion value can be provided
- Uses numerical model data assimilated with satellite and *in-situ* data

For forecasting of oil-spill trajectories over time, a user-friendly tool has been developed. This tool is intended to provide a quick capability for the user to forecast oil-spill trajectory over next 5 days. This tool uses numerical model surface forecast currents data. This numerical model is assimilated with satellite and *in-situ* data and operational at Space Applications Centre. Users can provide oil-spill area and date from past 15 days. User provided oil-spill area is seeded with the particles and the lagrangian particle trajectories are computed based on the surface currents and user defined lateral diffusion values. Future trajectories are also displayed over the future time-steps conveniently through GUI.

This new GUI tool is shown in **Figure 16** (left side of GUI) and highlighted in red box



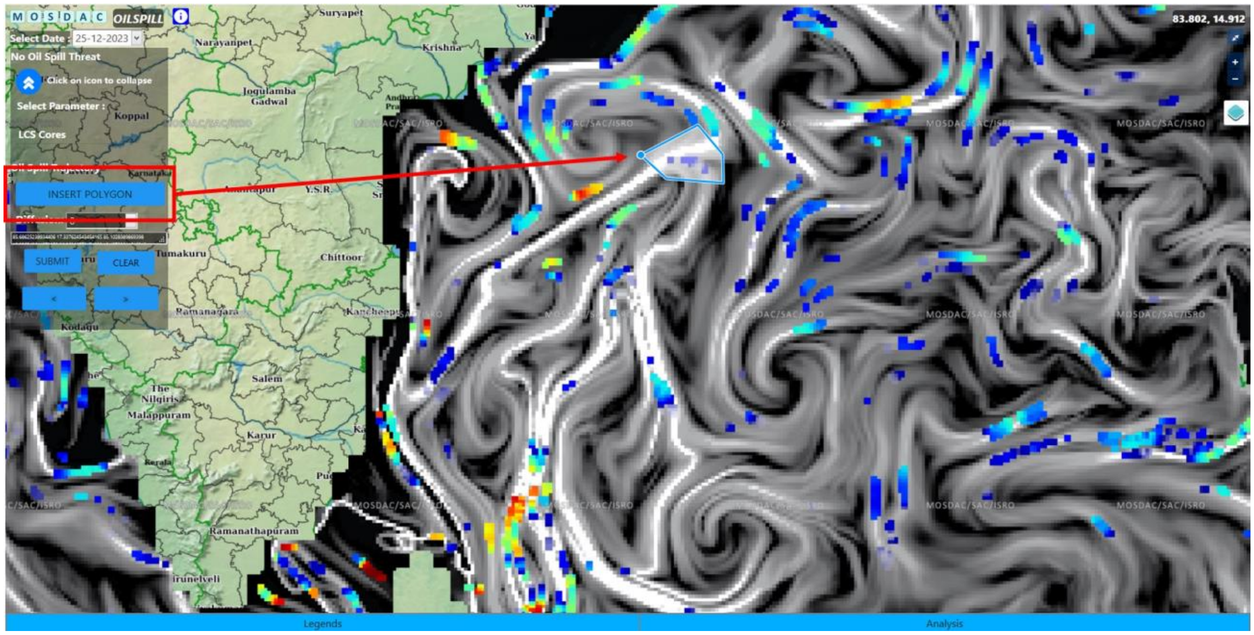
Select Date : 25-12-2023

Diffusion: 0

Figure 16: Oil-Spill trajectory forecasting tool in the GUI.

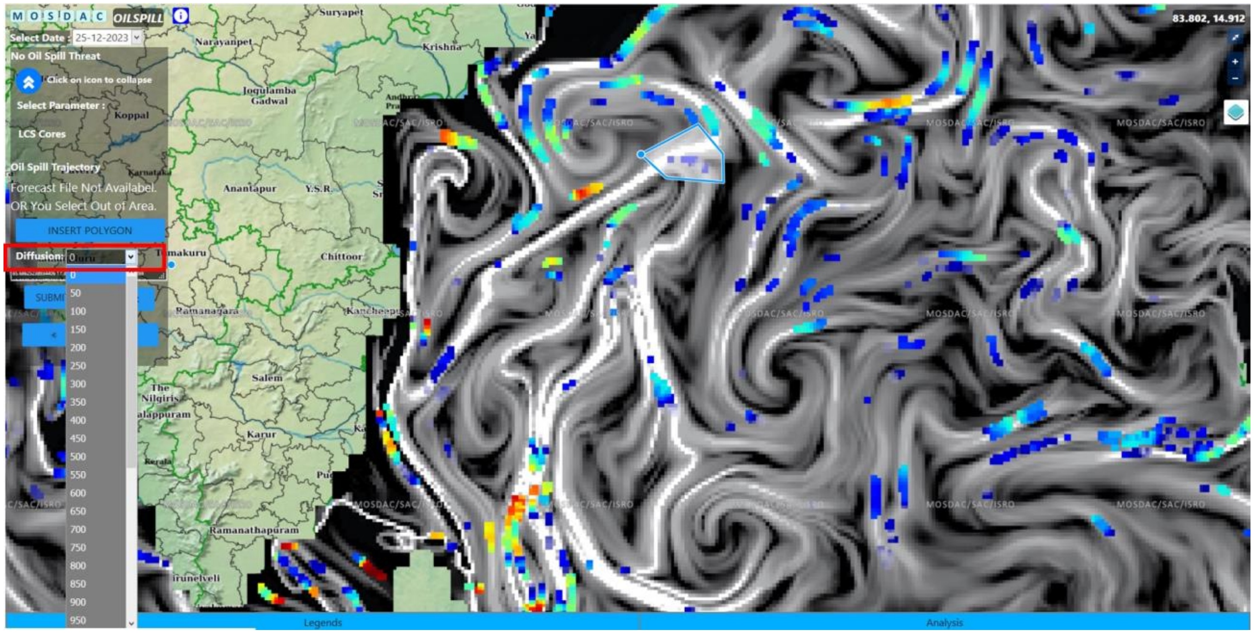
Users can enter the desired area using the following steps:

Step 1: Insert desired polygon



Select Date : 25-12-2023

Step 2: Insert the desired diffusion value in m^2/sec

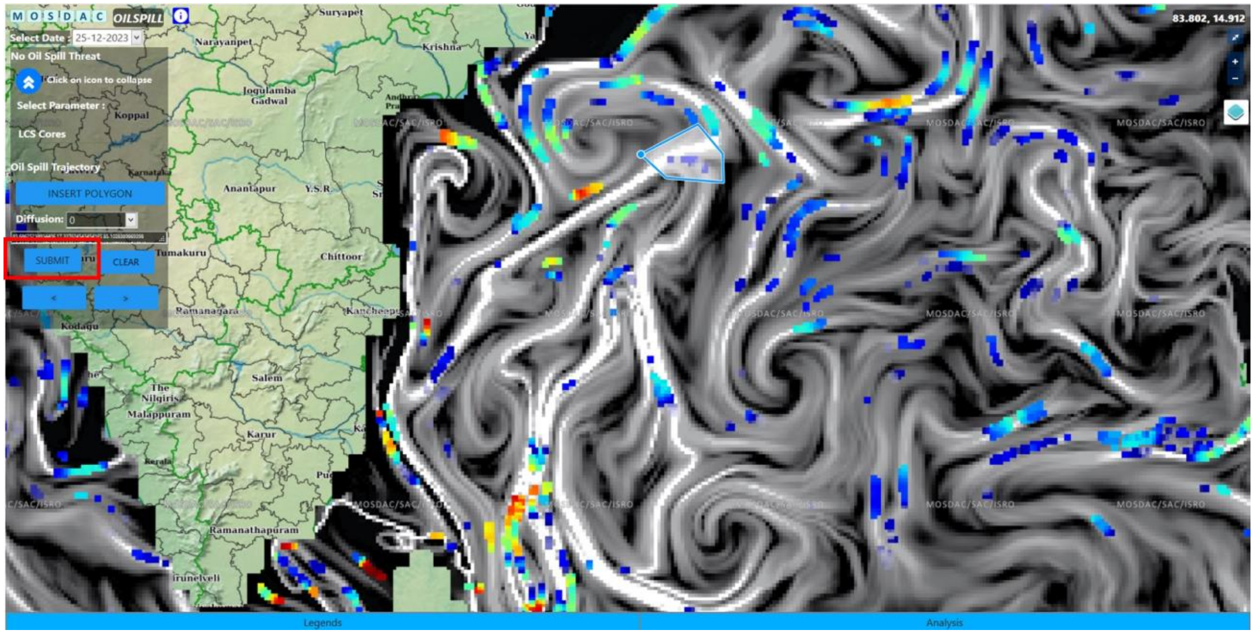


Select Date : 25-12-2023

Forecast File Not Availabel.
OR You Select Out of Area.

Diffusion: 0

Step 3: Insert the submit button



Select Date : 25-12-2023

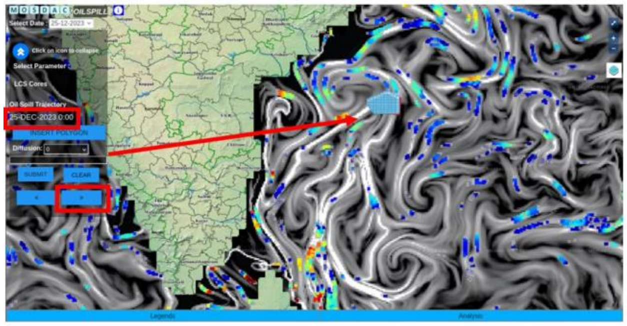
No Oil Spill Threat

Diffusion: 0

83.802, 14.912

Step 4: Get the forecasted oil-spill trajectories over different time-steps using arrow keys:

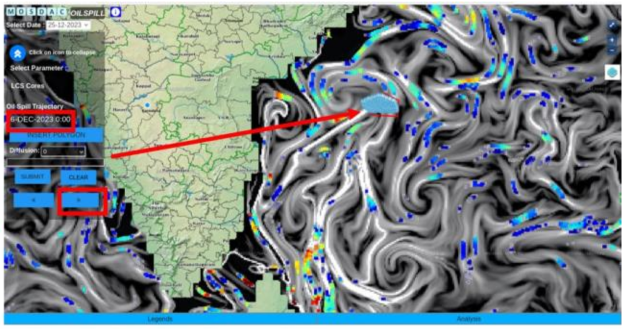
(Users can browse through various time-steps using the arrow keys)



Select Date : 25-12-2023

Oil Spill Trajectory: 25-DEC-2023 0:00

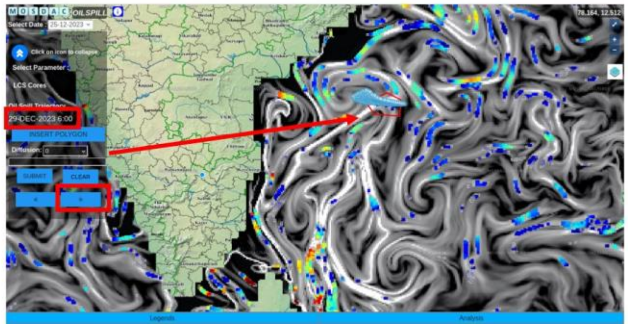
Diffusion: 0



Select Date : 25-12-2023

Oil Spill Trajectory: 6-DEC-2023 0:00

Diffusion: 0



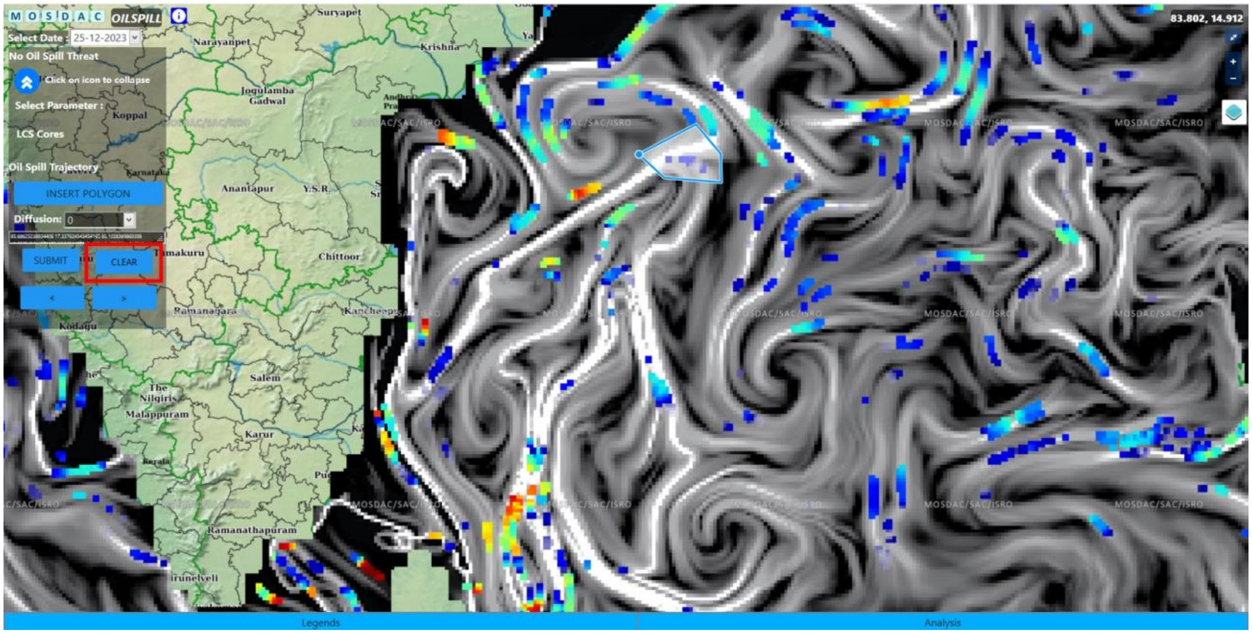
Select Date : 25-12-2023

Oil Spill Trajectory: 29-DEC-2023 6:00

Diffusion: 0

78.164, 12.512

Step 5: Reset and clear the GUI



Select Date : 25-12-2023

No Oil Spill Threat

Diffusion: 0

References

Olascoaga, María J., and George Haller. 2012. "Forecasting sudden changes in environmental pollution patterns." *Proceedings of the National Academy of Sciences* (National Acad Sciences). doi:10.1073/pnas.1118574109.

Onu, K., Florian Huhn, and George Haller. 2015. "LCS Tool: A computational platform for Lagrangian coherent structures." *Journal of Computational Science* (Elsevier) 7: 26-36. doi:10.1016/j.jocs.2014.12.0012.

Farazmand, Mohammad, and George Haller. 2012. "Computing Lagrangian coherent structures from their variational theory." *Chaos: An Interdisciplinary Journal of Nonlinear Science* (AIP) 22: 013128. doi:10.1063/1.3690153.